Crystal problems for binary systems

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This talk is based on joint works with Hans Knüpfer (University of Heidelberg), Florian Nolte (University of Heidelberg) and Mircea Petrache (PUC Chile).

In his paper "Über elektrostatische Gitterpotentiale", published in 1921, Max Born asked the following question related to ionic crystals: "How to arrange positive and negative charges on a simple cubic lattice of finite extent so that the electrostatic energy is minimal?". Considering periodic configurations of charges in one dimension, he proved the minimality of the alternation of charges -1 and +1.

In this talk, I will start by briefly explaining several one-dimensional crystallization results for binary systems of alternated charges and with three kind of interactions. Then, I will explain a connection between the translated lattice theta function and the optimal configuration of charges on a given lattice, when the interaction potential is completely monotone. Thus, a proof of Born's conjecture in any dimension, for orthorhombic lattices, will be given. Finally, we will see that the solution for the triangular lattice exhibits a surprising honeycomb structure.